Simplification of Mathematical Model of Double Fed Asynchronous Machine

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Abstract—Double Fed Asynchronous Machines in (DFAM) recent years have found wide application both as generators mainly in renewable energy sources (wind power, small hydropower), and as a motors embedded in various electric drives. In order to study the static and dynamic modes of operation of these machines as one of the most effective methods - the method of mathematical modeling is widely used. It was found that the algebraic-differential equations that make up the mathematical model of double fed induction machine are expediently represented in the d and q axes rotating with the rotor speed of the machine. Particularly this form allows relatively simple reproduction of control coordinates - amplitude and frequency converted to the rotor winding voltage. This mathematical model is based on the well-known Park equations for synchronous machines. It is proposed to simplify the algebraic-differential equations of the double-fed induction machine by reducing the transformer emf, the slip emf and the voltage drop on the stator winding of the machine. The results of calculations for the complete equations of the machine having the 6th order and on the simplified equations having the 4th order showed that the error in determining the regime parameters of the machine in steady-state conditions is in the range 0-7%, i.e. does not exceed 7 percent. The error in dynamic modes is also within acceptable limits. In the dynamics of the change in the regime parameters of DFIM with a sharp change in the disturbing influences slightly differ from each other, and with a sharp change in the control actions they make up 20-30%. This allows to recommend the proposed simplification for engineering calculations, especially when these machines operate in a group or in parallel with other machines and components of power systems.

Index Terms—Double Fed Asynchronous Machine; Complete Equations; Simplified Equations; Static and Dynamic Modes; Mathematical Model.

I. INTRODUCTION

Recently the double fed asynchronous machine, frequency controlled from the rotor side, are widely used both as generators (mainly in renewable energy sources) [1], [2] and as motors in electric drives of various mechanisms [3].

The advantages of this machine are well known – they are a relative simplicity and consequently increased reliability, a possibility of rotational frequency control in the range of 30-40%, a significantly limited value of frequency converter's power.

In recent years, they find also an application in the electrical equipment of sea-going vessels. They are used as

shaft generators, which allows maintaining the shaft generator's output parameters – voltage amplitude and its frequency within the permissible deviations when the rotational speed of main shaft deviates to one or another side by $\pm(15-20\%)$.

It should be noted that the modern frequency converters are performed on the basis of IGBT power transistors, or on fully controlled GTO thyristors, whose control circuit is constructed on sinusoidal pulse-width modulation (PWM – control) [4].

II. PROBLEM DEFINITION

It is known that mathematical simulation is one of the effective methods for studying of the static and dynamic characteristics of double fed asynchronous machines (DFAM). There are many researches devoted to study of steady-state and transient modes of DFAM, when they operate in wind power plants, for example [5], [6]. In them the differential equations, which form the base of mathematical model, are written, as a rule, in a complex form, then the real and imaginary parts of equations are separated. The temporal form of notation of DFAM equations in d, q axes, rotating with the machine's rotor speed, allowing a relative simple reproduction on the model of the amplitude and frequency of output voltage of frequency converter, supplying the machine's rotor winding, is presented in [7], [8]. In this case the mathematical model of DFAM given in [7], [8] is based on the well-known Park equations.

The purpose of this study is to assess the permissible simplifications of DFAM system's differential equations given in [7], [8], which form the basis of DFAM mathematical model, in order to recommend in the sequel the simplified mathematical model for studying of different operating modes of DFAM.

III. RESEARCH TECHNIQUE

As it has been noted, the mathematical model of DFAM is based on algebraic differential equations, written in d, q axes, rotating with a rotor speed of the machine [7]:

$$\left. \begin{aligned} P_s &= U_{ds} \cdot i_{ds} + U_{qs} \cdot i_{qs} = (-U_s \cdot \sin\theta) \cdot i_{ds} + (U_s \cdot \cos\theta) \cdot i_{qs} \\ P_r &= U_{dr} \cdot i_{dr} + U_{qr} \cdot i_{qr} = [-U_s \cdot k_{ar} \cdot \sin(k_p \cdot \tau)] \cdot i_{dr} + [U_s \cdot k_{ar} \cdot \cos(k_p \cdot \tau)] \cdot i_{qr} \\ P_{DFM} &= P_s + P_r \end{aligned} \right\} \quad (1)$$

Published on December 29, 2017.

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$$\text{where } k_s = \frac{x_r}{x_s \cdot x_r - x_m^2}; k_r = \frac{x_s}{x_s \cdot x_r - x_m^2};$$

$$k_m = \frac{x_m}{x_s \cdot x_r - x_m^2}.$$

It also needs to attribute to the output parameters of DFAM a rotational frequency of rotor $\omega_r = (1-s)$, as well as the active and reactive powers of the machine, which are determined by the following correlations:

$$\left. \begin{aligned} P_s &= U_{ds} \cdot i_{ds} + U_{qs} \cdot i_{qs} = (-U_s \cdot \sin\theta) \cdot i_{ds} + (U_s \cdot \cos\theta) \cdot i_{qs} \\ P_r &= U_{dr} \cdot i_{dr} + U_{qr} \cdot i_{qr} = [-U_s \cdot k_{ur} \cdot \sin(k_{fr} \cdot \tau)] \cdot i_{dr} + [U_s \cdot k_{ur} \cdot \cos(k_{fr} \cdot \tau)] \cdot i_{qr} \\ P_{DFM} &= P_s + P_r \end{aligned} \right\} \quad (2)$$

$$\left. \begin{aligned} Q_s &= U_{ds} \cdot i_{qs} - U_{qs} \cdot i_{ds} = (U_s \cdot \cos\theta) \cdot i_{qs} - (U_s \cdot \sin\theta) \cdot i_{ds} \\ Q_r &= U_{dr} \cdot i_{qr} - U_{qr} \cdot i_{dr} = [U_s \cdot k_{ur} \cdot \cos(k_{fr} \cdot \tau)] \cdot i_{dr} - [U_s \cdot k_{ur} \cdot \sin(k_{fr} \cdot \tau)] \cdot i_{qr} \\ Q_{DFM} &= Q_s + Q_r \end{aligned} \right\} \quad (3)$$

Algebraic – differential equations (1), (2), (3) form the complete mathematical model of double fed asynchronous machine. They are recorded in d, q axes, rotating with the rotor speed, and according to their structure they correspond to the Park equations for synchronous machines.

It is known, that when a synchronous machine is operating in a power system, its equations can be simplified by the mutual exclusion of transformer's emf and emf of slip in the stator circuits, and also a voltage drop on the active resistance of stator winding can be neglected in view of its smallness.

Let us also apply this method to double fed asynchronous machines, then first two equations of system (1) will take the form:

$$\left. \begin{aligned} \psi_{qs} &= U_s \cdot \sin\theta \\ \psi_{ds} &= U_s \cdot \cos\theta \end{aligned} \right\} \quad (4)$$

With taking into account (4) expression, the equations of DFAM after trivial transformations will simplify and take the form:

$$\left. \begin{aligned} p\Psi_{dr} &= U_{dr} - r_r(k_r \cdot \Psi_{dr} + k_m \cdot U_s \cdot \cos\theta) \\ p\Psi_{qr} &= U_{qr} - r_r(k_r \cdot \Psi_{qr} + k_m \cdot U_s \cdot \sin\theta) \\ p_s &= \frac{1}{T_j} \cdot m_{dr} - \frac{1}{T_j} \cdot k_m \cdot U_s (\Psi_{dr} \cdot \sin\theta - \Psi_{qr} \cdot \cos\theta) \\ p\theta &= s \end{aligned} \right\} \quad (5)$$

$$\text{where } U_{dr} = -U_s \cdot k_{ur} \cdot \sin(k_{fr} \cdot \tau);$$

$$U_{qr} = U_s \cdot k_{ur} \cdot \cos(k_{fr} \cdot \tau).$$

Thus, the system of differential equations (1), which has the 6th order, is reduced to a system of 4th order (5) as a result of simplifications and transformations.

IV. THE SIMULATION RESULTS

Thus, the matter of research is to compare the operation parameters of DFAM obtained by equations (1) with the results obtained by equations (5), and estimate their error.

The studies have been carried out on DFAM with parameters, expressed in the relative units: active stator resistances $r_s = 0.01$, rotor ones $r_r = 0.031$, inductances of magnetization $x_m = 4.8$, stator $x_s = 4.877$, rotor $x_r = 4.9$, inertia constant $T_j = 125$ rad., network voltage $U_s = 1$. The choice of system of base values and reduction of the parameters to relative units are well known [7], [8]. The calculated coefficients are: $k_s = 5.69$, $k_r = 5.66$, $k_m = 5.56$.

It is necessary to pay attention to one else circumstance. The control coordinates of rotor voltage components U_{dr} and U_{qr} include a factor k_{ur} , which takes into account a control of amplitude of voltage, supplied to a rotor winding of DFAM, and a k_{fr} factor, taking into account a frequency control of this voltage. In this process, since a factor U_s stands before these factors – it means, that the stator (network) voltage U_s is supplied without additional transformation to a frequency converter, feeding a rotor winding of DFAM. In the presence of a rotor transformer it is natural, that $U_r = k_{tr} \cdot U_s$ – where U_r is a voltage at the output of the auxiliary transformer, supplying the frequency converter, k_{tr} is a transformation ratio of this transformer.

And finally, as it is known in a real double fed machine a transition from control downwards from the synchronous speed, to control upwards from the synchronous speed is carried out by permutation of two phases' supply of three-phase rotor winding. Since a three-phase machine is reduced in mathematical model to a two-phase one, this position is simulated by changing of a sign before voltage at one of the axes.

In the recorded (1) and (5) equations $U_{dr} = -U_s \cdot k_{ur} \cdot \sin(k_{fr} \cdot \tau)$ and $U_{qr} = +U_s \cdot k_{ur} \cdot \cos(k_{fr} \cdot \tau)$, and this corresponds to rotational frequency change of DFAM shaft from ~ 1 to 0.85 when $k_{ur} = k_{fr}$ change in the range of from ≈ 0 to 0.15. To control a rotational frequency of DFAM in the range of from 1 to 1.15, it is necessary to change the sign, for example, before U_{qr} , i.e. to present it in the form $U_{qr} = -U_s \cdot k_{ur} \cdot \cos(k_{fr} \cdot \tau)$. Or, to leave the system of equations (1) and (5) in an unchanged form, it is possible to operate with negative and positive values of control factors: for example, the negative values of $k_{ur} = k_{fr} = -0.15$ factors will correspond to control upwards from the synchronous speed ($\omega_r = 1.15$), and the positive values $k_{ur} = k_{fr} = 0.15$ will correspond to control downwards from the synchronous speed ($\omega_r = 0.85$). This technique is equivalent to change of a sign before one of the rotor voltage components.

On the fluktoqrammas Fig. 1 the curves of change of DFAM operation parameters when simulating on the complete system equations (1) are presented. In Fig. 1 (a) the fluktoqramma of change of electromagnetic torque m_{em} is presented. A direct startup of DFAM with short-circuited rotor windings of the machines i.e. at $U_{dr} = U_{qr} = 0$ is carried out in the range of from $\tau = 0$ to $\tau = 1000$ radian, in this process a value of driving torque is equal to $m_{dr} = -0.6$ (generator mode). In this range a DFAM rotation frequency varies from $\omega_r = 0$ to $\omega_r \approx 1.02$ (Fig. 1,b). That is, a rotational frequency is more than 1, which also indicates the generator mode. At 1000th rad. a voltage is supplied to a rotor winding, the amplitude and frequency of which are

equal to $k_{ur} = k_{fr} = -0.15$, in this process ω_r (Fig. 1,b) changes from $\omega_r \approx 1.02$ to $\omega_r = 1.15$. The active power of stator winding p_s remains constant and equal to $p_s = -0.595$ (Fig. 1, c), and rotor winding after 1000 rad. becomes equal to $p_r = -0.0667$ (Fig. 1,e). Accordingly, the reactive powers of windings also changes: q_s changes from the value $q_s = 0.297$ (plus sign indicates that power is consumed from the network) to $q_s = -0.38$ (power is output to the network); reactive power of the rotor winding after 1000 rad. becomes equal to $q_r = -0.111$ (Fig. 1,f). In accordance with the expressions (2) and (3) the total active and reactive powers of the machine are accordingly equal to $p_{DFM} = -0.661$ and $q_{DFM} = -0.492$ (Fig. 1) with a rotational frequency $\omega_r = 1.15$.

Fig. 1. Fluctograms of the change in the operational parameters of DFAM in the simulation by the complete equations of the system under control $k_{ur} = k_{fr} = -0.15$

The fluktoqrammas of change of the same machine's parameters, obtained as a result of solving of simplified system's equations (5) are presented in Fig. 2 (a, b, c, d, e, f, g, h). In Table I a comparative analysis of these parameters is given, and the errors in steady-state modes are determined (the parameters' values before control are given in the numerator (up to 1000 rad.), and in the denominator - the ones after control (after 1000 rad.)).

TABLE I: THE RESULTS OBTAINED FROM THE COMPLETE AND SIMPLIFIED EQUATIONS FOR $k_{ur} = k_{fr} = -0.15$

Mode's parameters (steady-state values)	Calculation by the complete equations	Calculation by simplified equations	Error, %
ω_r	1.0193/1.15	1.0195/1.15	-0/0
m_{em}	-0.6/-0.6	-0.6/-0.6	0/0
p_s	-0.595/-0.595	-0.6/-0.6	0.84/0.84
p_r	0/-0.0667	0/-0.0654	0/1.9
q_s	0.296/-0.38	0.295/-0.407	0.33/7
q_r	0/-0.111	0/-0.116	0/4.5
p_{DFM}	-0.595/-0.661	-0.6/-0.665	0.84/0.6
q_{DFM}	-0.297/-0.492	-0.295/-0.523	0.67/6.3

TABLE II. THE RESULTS OBTAINED FROM THE COMPLETE AND SIMPLIFIED EQUATIONS FOR $k_{ur} = 0.185$ AND $k_{fr} = 0.15$

Mode's parameters (steady-state values)	Calculation by the complete equations	Calculation by simplified equations	Error, %
ω_r	1.0193/0.85	1.0195/0.85	~0/0
m_{em}	-0.6/-0.6	-0.6/-0.6	0/0
p_s	-0.595/-0.594	-0.6/-0.6	0.84/0.85
p_r	0/0.115	0/0.117	0/1.74
q_s	0.296/-0.436	0.295/-0.462	0.33/5.9
q_r	0/0.122	0/0.126	0/3.28
p_{DFM}	-0.595/-0.479	-0.6/-0.483	0.84/0.84
q_{DFM}	0.297/-0.314	0.295/-0.336	0.67/7

Comparison of the calculation results when rotational frequency control both upwards from the synchronous speed (Table I), and downwards from the synchronous speed (Table II) shows, that the maximum error in both cases is obtained when determining the reactive power of machine (about 7%). Bearing in mind, that the parameters of machine itself (active resistance and inductive reactance) are determined with an error of about 10%, such error when replacing a system of complete differential equations of a mathematical model by simplified one, is completely acceptable for engineering calculations.

To evaluate the interactive parameters, a dynamic mode was reproduced, consisting in a sharp 2 times increase of driving torque on machine's shaft, i.e. stepwise change from $m_{dr} = -0.6$ to $m_{dr} = -1.2$. Besides of that, to trace visually the transient process, the inertial constant of system T_j has also been 2 times increased from $T_j = 125$ radian to $T_j = 250$ radian.

Fig. 2. Fluctograms of the change in the DFIM operational parameters as per simulation using simplified equations of the system
 $k_{ur} = k_{fr} = -0.15$

On the fluctograms Fig. 3 the dynamics of change of DFAM operation parameters (torque m_{em} , rotational frequency ω_r , active power of stator p_s , reactive power of stator q_s , active and reactive powers of rotor p_r and q_r and total powers of machine p_{DFM} and q_{DFM}) has been presented when simulation on the complete machine's equations (system of equations (1)). Here in the range of from 0 to 500 rad. the asynchronous startup of the machine with output it to a generator mode has been carried out. In this range at the initial stage the torque values m_{em} , active power of stator p_s , reactive power of stator q_s , as well as the total active and reactive powers have periodic components.

On the fluctograms Fig. 4, where the dynamics of change of DFAM operation parameters, calculated on the simplified simulation scheme for the same time range is presented, the periodic components in the curves above the

noted curves are absent, but the times of output to steady-state value for these curves practically coincide.

Fig. 3: Dynamic pattern of the operation parameters of the DFIM as per simulation using complete equations of the machine at $k_{ur} = k_{fr} = -0.15$

Fig. 4. Dynamic pattern of the operation parameters of the DFIM as per simulation using simplified equations of the machine at $k_{ur} = k_{fr} = -0.15$

In Fig. 3 and Fig. 4 control activates from the time period of 500 rad, i.e. the rotor circuits open and voltage is supplied there $k_{ur} = k_{fr} = -0.15$. Comparing the corresponding curves Fig. 3 and Fig. 4 it should be noted, that the dynamics of control effects on variables as follows: in simplified equations the first spikes of variables in amplitude exceed the ones of corresponding variables when simulating on the total equations by 20-30%, but the number of oscillations up to the steady-state value for these variables differ slightly.

And finally, within the period of from 1500 rad. to 2000 rad. the torque on DFAM shaft has step-wise increase up to the value $m_{dr} = -1.2$ (i.e. 2 times). In this process, a comparison of behavior of operation parameters' dynamics, when surge and drop of this torque, shows that all the operation parameters according to amplitude and number of oscillations practically differ insignificantly from each other. This indicates that the dynamics of change of perturbation parameters are practically the same both for studies on the complete equations of systems and when study on the presented simplified equations.

In conclusion, it should be noted, that the relay (stepwise) changes of control parameters (k_{ur} and k_{fr}) and perturbation actions (m_{dr}) is a theoretically possible principle of influence, used in automatic control systems, in real systems both the control parameters and external disturbances cannot step-wisely change, and therefore some differences in the form of curves of transients when study on complete and simplified equations will be leveled.

V. CONCLUSION

The method of simplification of complete algebraic-differential equations, constituting the basis of mathematical model of frequency-controlled double fed asynchronous machines, finding the wide application both as generators and as motors, is proposed. This simplification allows reducing the order of the equations (from the 6th order to 4th one) and simplifying its structure.

The legitimacy of such simplification is proved - the steady-state values of DFAM operation parameters vary in the range of 0-7%, which is quite acceptable for engineering calculations, especially as the initial values of machine's parameters are determined with an accuracy of no more than 10%.

The dynamic components of operation parameters when stepwise change of control coordinates of the machine (k_{ur} and k_{fr}) in the first oscillation can differ considerably one from another in amplitude, but the transient time and number of oscillations are practically insignificantly different from each other.

The same components of operation parameters with stepwise change of perturbation actions (m_{dr}), practically in amplitude and number of oscillations differ insignificantly from each other when studying on complete and simplified equations.

Thus, such simplification of the equations and their representation in the form of mathematical model of DFAM is recommended to be carried out when they operate in an electrical system in combination with electrical networks and other generators, as well as in calculations where it is necessary to determine only the steady-state values of operation parameters of the machine.

REFERENCES

- [1] Windenergie 2006. Herausgeber (BWE – Service GmbH. 2006.
- [2] R. I. Mustafayev, L. H. Hasanova, M. M. Musayev, E. M. Mamedov, Ch. I. Nabyev. Modeling and analysis of operating regimes of hydraulic units of small hydropower plant with double fed machine. *Scientific and Technical Journal Electromechanics*, no 6, pp. 59-66, 2015.
- [3] A. V. Aristov. Performance characteristics of the electric drive of oscillating movement with a double-feed machine. *Bulletin of the Tomsk Polytechnic University*. no 3, pp. 101-107, 2003.
- [4] Dejan Schreiber. High Power Converter for Renewable Energy Sources. *Power electronics*, no 5, pp. 90-94, 2010.
- [5] S. Müller, M. Deicke, Rik W. De Doncker. Doubly fed induction generator systems for Wind Turbines. *IEEE Industry applications magazine*. pp. 26-33, May/june 2002.
- [6] J. L. Da Silva, R.G. de Oliveira, S.R. Silva, B. Rabelo and W. Hofmann. A Discussion about a Start-up Procedure of a Doubly-Fed Induction Generator System. *Norpie/2008. Nordic Workshop on Power and Industrial Electronics*. June 9-11, 2008.
- [7] R. I. Mustafayev, L. H. Hasanova. The dynamic and static modes modeling of wind power plant operation with double-fed asynchronous machine. *Elektrotehnika*, no 9, pp. 11-15, 2008.
- [8] R. I. Mustafayev, L. H. Hasanova. Simulation and investigation of quasi-steady operating conditions of wind power installations equipped with frequency-controlled induction generators. *Electricity*, No 6, pp. 36-41, 2009

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